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(2) in the equation to the plane gives,

$$x'=\frac{a^2x(c-z)}{c(x^2+y^2)},\quad y'=\frac{a^2y(c-z)}{c(x^2+y^2)}.$$

These values of x', y' in (1), $x^2+y^2-(a^2/c^2)(c-z)^2=0$, a cone of revolution as the locus.

* * *

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

49. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

A rectangular stick of timber of known dimensions is placed upon a platform of given height in a vertical position with the center above the edge of platform, and slightly displaced from the vertical. Where and in what manner will it strike the ground?

I. Solution by the PROPOSER.

Any body rotating about the center of an end has the energy,

$$\tfrac{1}{3}(\omega^2)m(a^2+b^2).$$

If the body has fallen through the angle θ, the energy is

$$\tfrac{1}{2}b(1-\cos\theta)m.$$

$$\therefore\ 2\omega^2(a^2+b^2)=3b(1-\cos\theta)+\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

The body will leave the platform when the statical pressure=centrifugal force. The pressure$=m\cos\theta$. Centrifugal force$=\tfrac{1}{2}m\omega^2b$.

$$\therefore\ \omega^2b=2\cos\theta\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

From (1) and (2), $\omega=\sqrt{\dfrac{6b}{4a^2+7b^2}}$ .... (3), and $\cos\theta=\dfrac{3b^2}{4a^2+7b^2}$ ....(4).

Take the edge of platform as origin. Let the axis of x be horizontal and the axis of y vertical. Resolve the angular velocity of the center of gravity into its vertical and horizontal components at the instant of the stick leaving the platform.

$$\left.\begin{aligned}V_x&=\tfrac{1}{2}b\omega\cos\theta\\ V_y&=\tfrac{1}{2}b\omega\sin\theta\end{aligned}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5).$$

For the acceleration we have $\frac{d^2x}{dt^2}=0,\qquad \frac{d^2y}{dt^2}=-g.$

Then $\frac{dx}{dt}=c$, and $\frac{dy}{dt}=-gt+c_2$.

Let us begin to reckon time from the instant that the body leaves the platform.

Then $\frac{dx}{dt}=V_x$, and $\frac{dy}{dt}=-gt+V_y$.

$x=V_xt+c_3$. $y=-\frac{1}{2}gt^2+V_yt+c_4$.

When $t=0$, $x=\frac{1}{2}b\sin\theta$, and $y=\frac{1}{2}b\cos\theta$.

$$\left.\begin{array}{l}\text{Then } x=V_xt+\frac{1}{2}b\sin\theta\\ \text{and } y=-\frac{1}{2}gt^2+V_yt+\frac{1}{2}b\cos\theta\end{array}\right\}\dots\dots\dots\dots\dots\dots(6).$$

These give the motion of the center of gravity.

Call T the time taken for one end to reach the ground. Then after leaving the platform it will have rotated through the angle $T\omega$.

It therefore makes an angle $\theta+T\omega$ with the vertical.

The center of gravity has fallen,

$$Y_1=-\frac{1}{2}gT^2+V_yT+\frac{1}{2}b\cos\theta.\quad \text{Also } X_1=V_xT+\frac{1}{2}b\sin\theta.$$

The center of gravity will be the distance $\frac{1}{2}b\cos(\theta+T\omega)$ from the ground.

Now $Y_1+\frac{1}{2}b\cos(\theta+T\omega)=H$, the height of tower.

Or, $\frac{1}{2}b\cos(\theta+T\omega)-\frac{1}{2}gT^2+V_yT+\frac{1}{2}b\cos\theta=H$..........................(7).

From this equation T may be determined. The horizontal distance from the foot of the tower will then be given by the equation,

$$X=X_1+\frac{1}{2}b\cos(\theta+T\omega)=V_xT+\frac{1}{2}b\sin\theta+\frac{1}{2}b\sin(\theta+T\omega).$$

II. Solution by **ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O. University, Mississippi.**

The stick turns about its lower extremity until it reaches a horizontal position with an angular velocity given by $\omega^2=(3g/2a)$, $2a$ being its length.

Subsequently there are two motions which may be considered independently. One is that of rotation about the center of gravity with a constant angular velocity, ω; the other, that of translation, the center of gravity falling vertically with an initial velocity, $a\omega$.

Estimating the motion from the horizontal position, the stick is vertical when it has turned through an odd number of right angles; that is, at the end of $n\pi/2\omega$ seconds, n being any odd number. If S_v denote the distance from the level of the platform to the lowest point of the stick at the instants of verticality, the motion of translation gives,

$$S_v - a = a\omega\frac{n\pi}{2\omega} + \tfrac{1}{2}g\left(\frac{n\pi}{2\omega}\right)^2,$$

or, substituting the value of ω,

$$S_v = \{1 + \tfrac{1}{2}(\pi n)[1 + \tfrac{1}{6}(\pi n)]\}a, \text{ } n \text{ being odd.}$$

Similarly the positions of horizontality are given by

$$S_h = \tfrac{1}{2}(\pi n)[1 + \tfrac{1}{6}(\pi n)]a, \text{ } n \text{ being even.}$$

If any value of S_v or S_h equals D, the distance from the platform to the ground, the stick will strike the ground, in the one case vertically, in the other, horizontally.

The discussion might be continued in general terms. Instead of this, however, let $a=1$ foot, and $D=10$ feet.

Giving to n the values of 1, 2, and 3 in the proper equations, the first and second values of S_v are found to about 3.4 and 13.1, and the first value of S_h about 6.4. Consequently the stick will strike the ground in passing from a horizontal toward a vertical position.

Since in falling 6.4 feet a half revolution has been made, the time for this motion is π/ω seconds, and the velocity of the center of gravity when the stick is horizontal for the last time is $\omega + g(\pi/\omega)$, remembering that $a=1$. If the stick turns through an angle θ before striking the ground, the center of gravity falls through $(3.6 - \sin\theta)$ feet in θ/ω seconds, giving the equation,

$$3.6 - \sin\theta = [\omega = g(\pi/\omega)](\theta/\omega) + \tfrac{1}{2}g[(\theta/\omega)]^2,$$

which reduces to $3.6 - \sin\theta = 3.1\theta + \tfrac{1}{3}\theta^2$, approximately ; from which $\theta = 48° \, 20'$, about.

The horizontal distance from the edge of the platform to the point at which the stick touches the ground is $1 + \cos\theta$, or 1 foot, 8 inches, approximately.

θ is, of course, the inclination of the stick to the horizontal at the instant of contact with the ground.

In the last part of this work the thickness of the stick has been neglected.